# Digital Communication Systems
## EES 452

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## 6-7. Waveform Channel and The Conversion to Vector Channel

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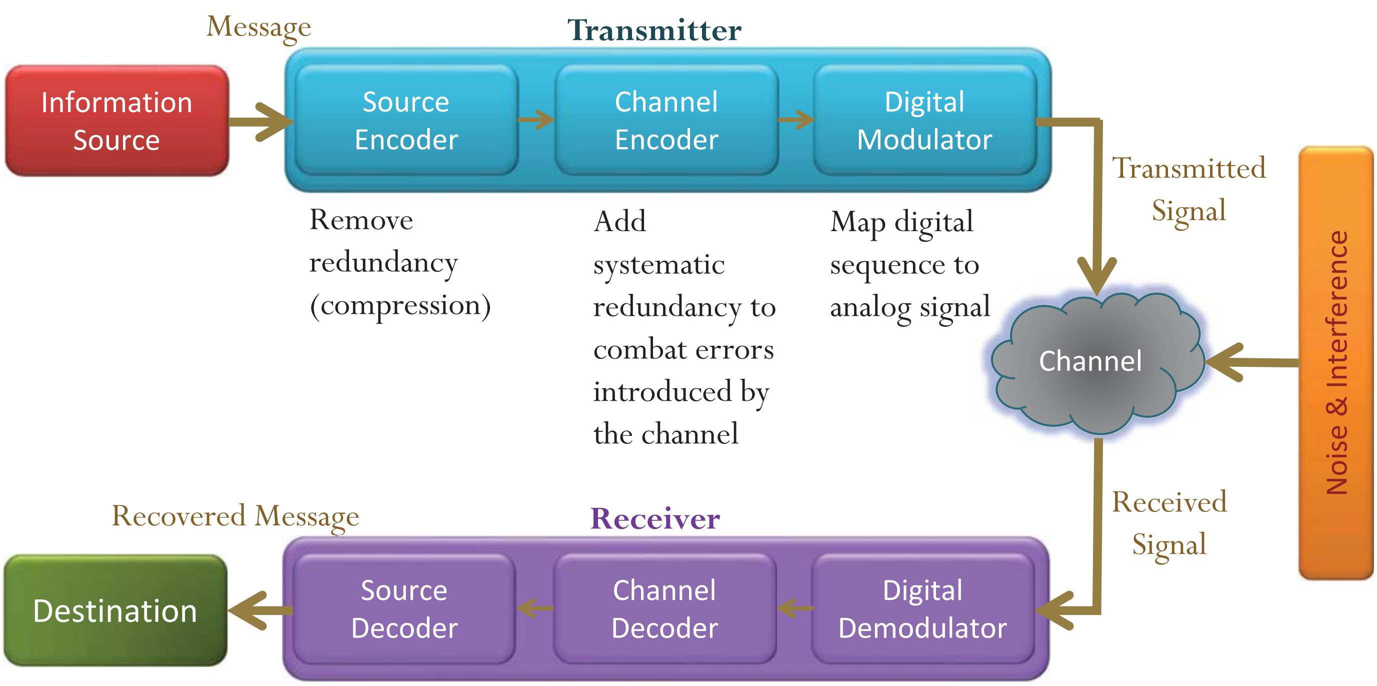
# Elements of digital commu. sys.

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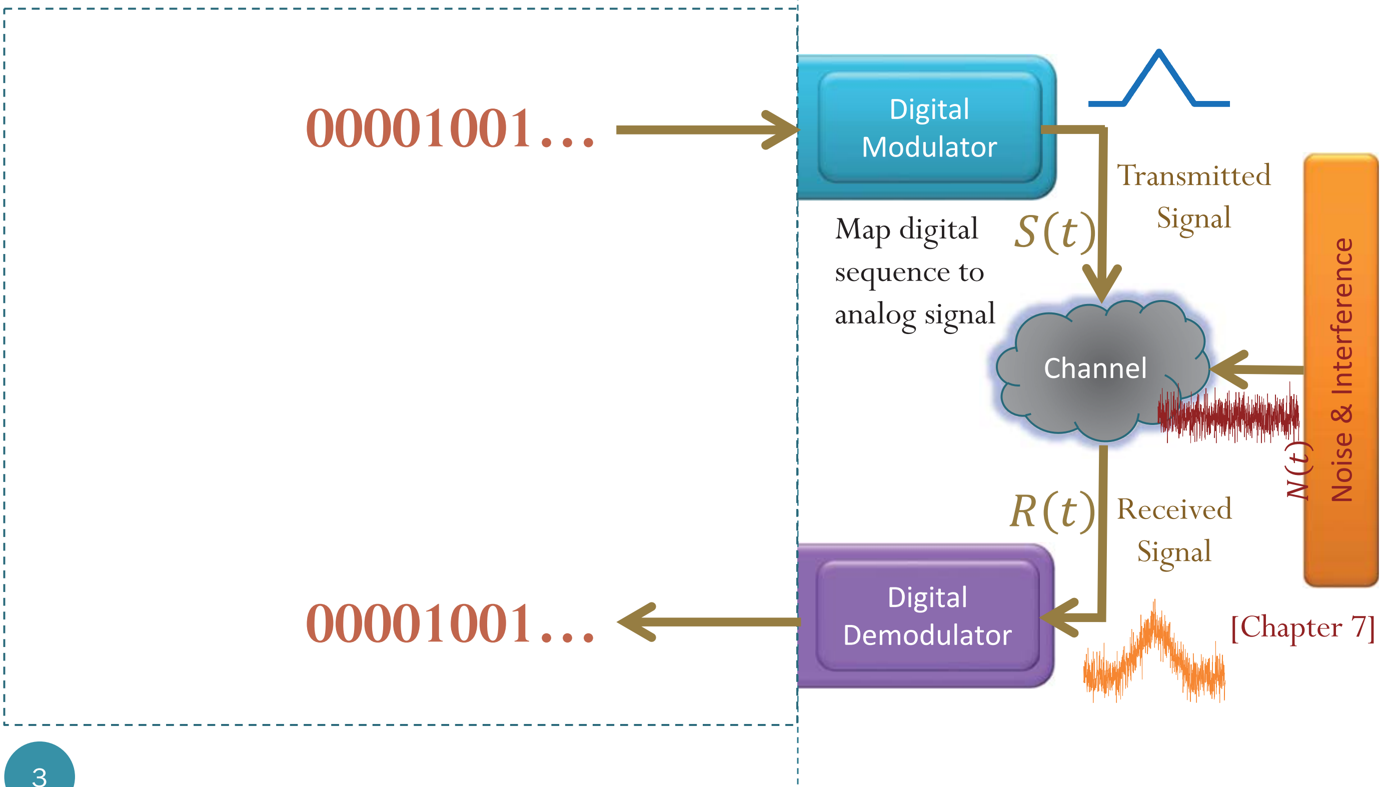
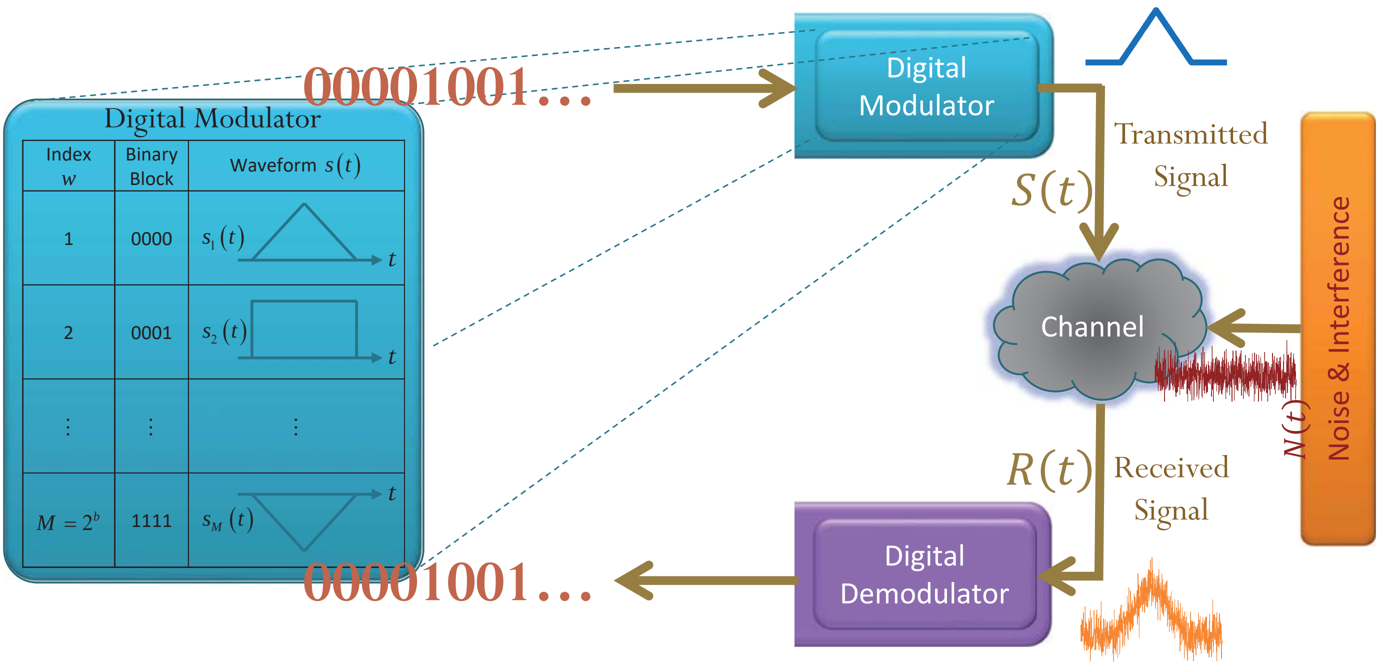
# Digital Modulation/Demodulation



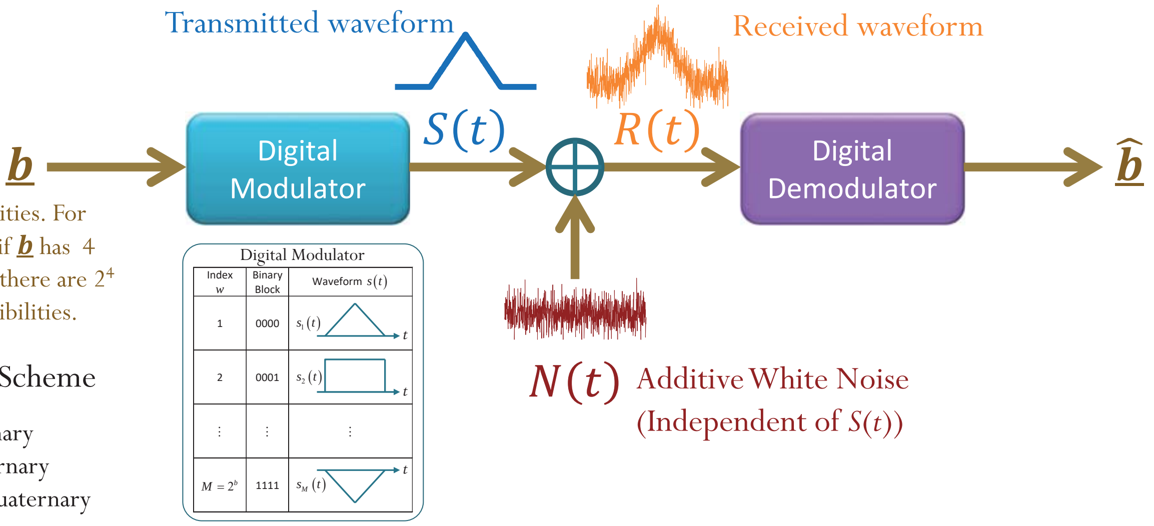



# Digital Modem: Ex 1

$b = 4$: Work on 4-bit blocks. Need $M = 2^b = 16$ different waveforms to represent different possibilities of the bit blocks.



# Analysis of Digital Modem




$M$ possibilities. For example, if $b$ has 4 bits, then there are $2^4$ = 16 possibilities.

**$M$-ary** Scheme

$M$ =2: Binary
$M$ = 3: Ternary
$M$ = 4: Quaternary

$M$ possible "messages" requires $M$ possibilities for $S(t)$:

$$\{s_1(t), s_2(t), \ldots, s_M(t)\}$$ ← We refer to this as the **signal set**

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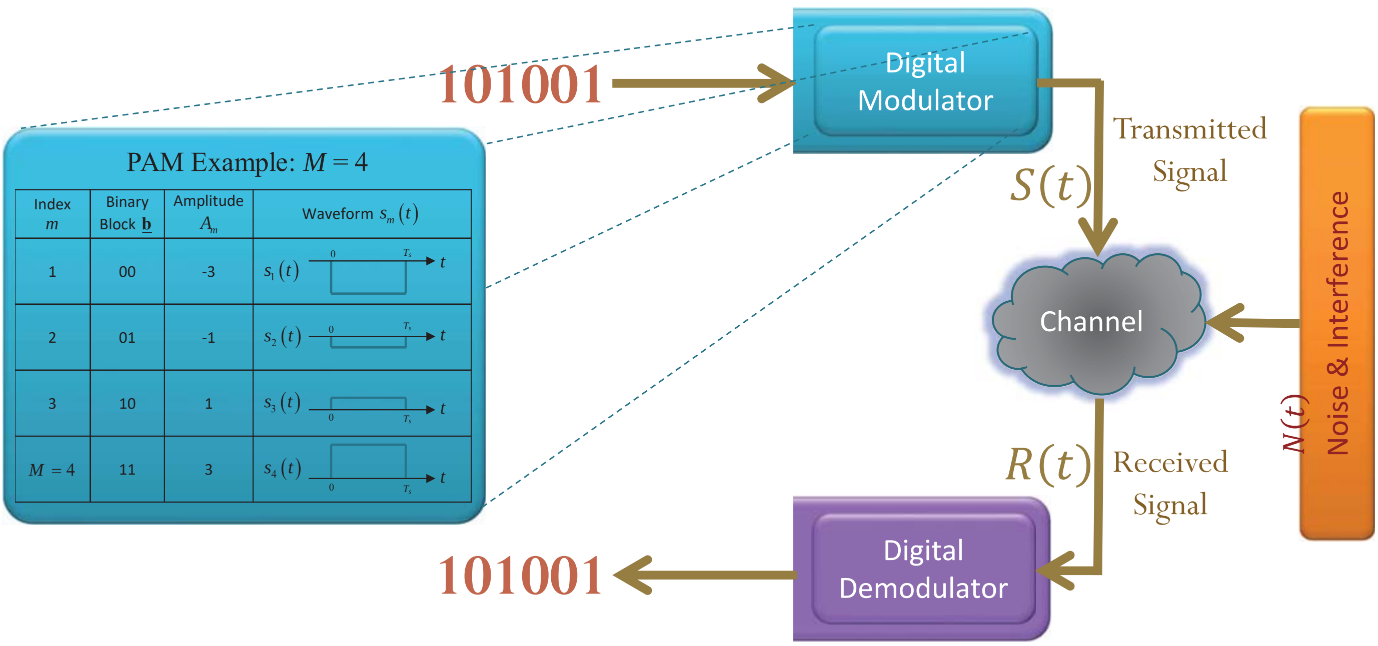
# Digital Modem: Ex 2

$b = 2$: Work on 2-bit blocks. Need $M = 2^b = 4$ different waveforms to represent different possibilities of the bit blocks.




| Index $m$ | Binary Block **b** | Amplitude $A_m$ | Waveform $s_m(t)$ |
|---|---|---|---|
| 1 | 00 | -3 | $s_1(t)$ |
| 2 | 01 | -1 | $s_2(t)$ |
| 3 | 10 | 1 | $s_3(t)$ |
| $M$ = 4 | 11 | 3 | $s_4(t)$ |

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# Pulse Amplitude Modulation

- Use a common **pulse $p(t)$**.
  - **Amplitude-Shift Keying (ASK)**: $p(t) = g(t)\cos(2\pi f_c t)$
    - $E_p = \frac{1}{2}E_g$ when $g$ is an energy signal that is bandlimited to $B < f_c$
- The pulse is scaled by $M$ different "amplitudes":

$$s_m(t) = A_m p(t), \quad 1 \le m \le M$$

$p(t)$


PAM Example: $M = 2$

| Index $m$ | Binary Block $\mathbf{b}$ | Amplitude $A_m$ | Waveform $s_m(t)$ |
|---|---|---|---|
| 1 | 0 | -1 | $s_1(t)$ |
| $M = 2$ | 1 | 1 | $s_2(t)$ |

PAM Example: $M = 4$

| Index $m$ | Binary Block $\mathbf{b}$ | Amplitude $A_m$ | Waveform $s_m(t)$ |
|---|---|---|---|
| 1 | 00 | -3 | $s_1(t)$ |
| 2 | 01 | -1 | $s_2(t)$ |
| 3 | 10 | 1 | $s_3(t)$ |
| $M = 4$ | 11 | 3 | $s_4(t)$ |



# Analysis of Digital Modem

$\{s_1(t), s_2(t), \dots, s_M(t)\}$

**Waveform Channel**: $R(t) = S(t) + N(t)$

Find "orthonormal basis" $\{\phi_1(t), \phi_2(t), \dots, \phi_K(t)\}$

Inner Product:
For two waveforms $x(t)$ and $y(t)$,

$$\langle x(t), y(t)\rangle = \int_{-\infty}^{\infty} x(t)y^*(t)\,dt$$

$$\begin{pmatrix}\langle R(t),\phi_1(t)\rangle\\ \langle R(t),\phi_2(t)\rangle\\ \vdots\\ \langle R(t),\phi_K(t)\rangle\end{pmatrix} \quad \begin{pmatrix}\langle S(t),\phi_1(t)\rangle\\ \langle S(t),\phi_2(t)\rangle\\ \vdots\\ \langle S(t),\phi_K(t)\rangle\end{pmatrix} \quad \begin{pmatrix}\langle N(t),\phi_1(t)\rangle\\ \langle N(t),\phi_2(t)\rangle\\ \vdots\\ \langle N(t),\phi_K(t)\rangle\end{pmatrix}$$

Ex:
$K = 1$: PAM, ASK
$K = 2$: PSK, QAM

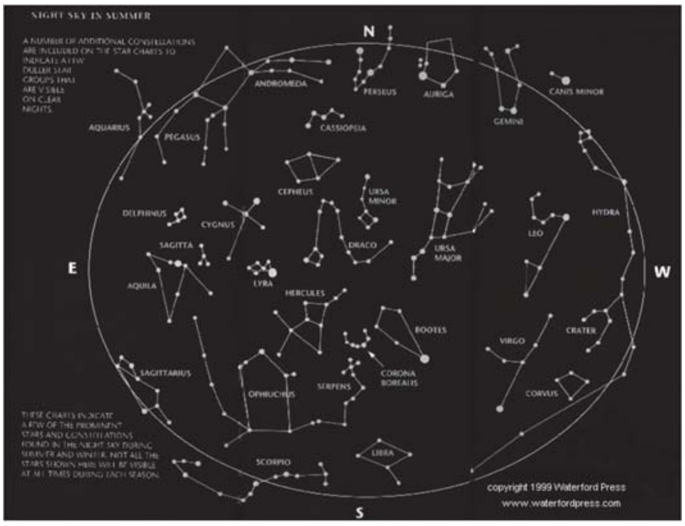
**Vector Channel:** $\vec{\mathbf{R}} = \vec{\mathbf{S}} + \vec{\mathbf{N}}$

$\in \left\{\vec{\mathbf{s}}^{(1)}, \vec{\mathbf{s}}^{(2)}, \dots, \vec{\mathbf{s}}^{(M)}\right\}$

can be visualized in the form of **signal constellation**

$\vec{\mathbf{R}}$, $\vec{\mathbf{S}}$ and $\vec{\mathbf{N}}$ are all random vectors.
Each vector contain $K$ random variables.

# Star Constellations

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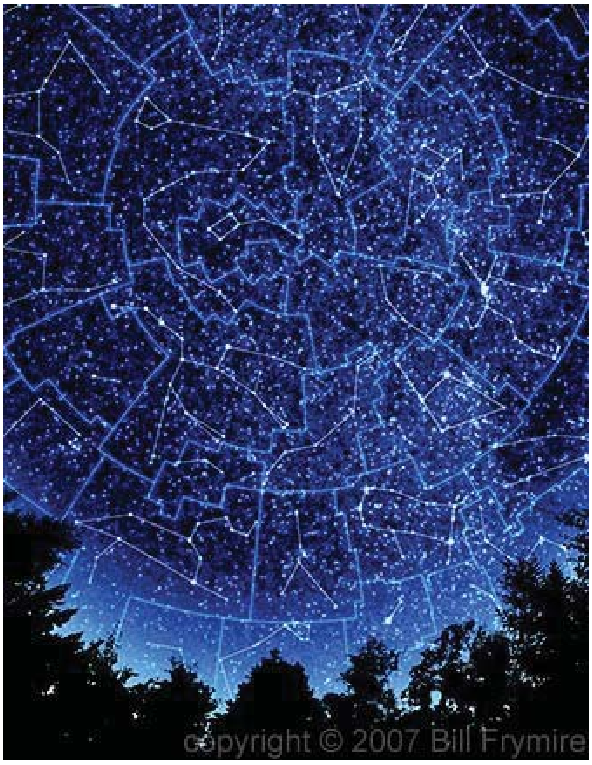



[http://68.media.tumblr.com/89eed4669ec511bbb6413acb8da8b0e2/tumblr_mz3qmyQeLH1rhb9f5o1_r1_400.jpg]



# Star Constellations

## constellation

***noun*** [ C ]

**UK** / ˌkɒn.stəˈleɪ.ʃᵊn/ **US** / ˌkɑːn.stəˈleɪ.ʃᵊn/

**any of the groups of stars in the sky that seem from earth to form a pattern and have been given names**

sololos/E+/GettyImages

LONGDO Dict
บริการค้นหาและร่วมสร้างเนื้อหาพจนานุกรมหลายภาษา-ไทย

**English-Thai: NECTEC's Lexitron-2 Dictionary [with local updates]**

| | |
|---|---|
| **constellation** | [N] กลุ่ม |
| **constellation** | [N] กลุ่มดาว, **Syn.** group of stars, configuration of stars |

**English-Thai: HOPE Dictionary [with local updates]**

| | |
|---|---|
| **constellation** | n. กลุ่มดาว, **See also:** constellatory adj. |

**English-Thai: Nontri Dictionary**

| | |
|---|---|
| **constellation** | (n) หมู่ดาว,ดาวฤกษ์,ดารากร |

**อังกฤษ-ไทย: คลังศัพท์ไทย โดย สวทช.**

| | |
|---|---|
| **constellation** | กลุ่มดาว, กลุ่มของดาวฤกษ์ที่ปรากฏในท้องฟ้า เคลื่อนที่ไปพร้อมกันเป็นกลุ่ม ๆ เช่นกลุ่มดาวจระเข้ กลุ่มดาวเต่า เป็นต้น [พจนานุกรมศัพท์ สสวท.] |

https://dictionary.cambridge.org/dictionary/english/constellation
https://dict.longdo.com/search/constellation

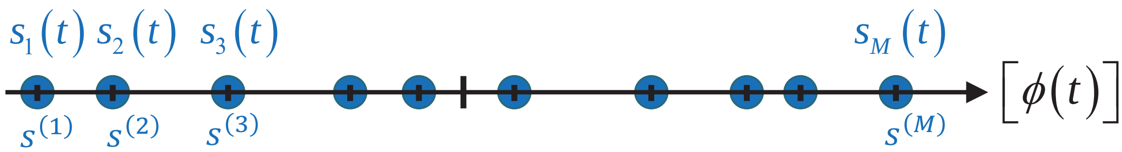
# Constellation for PAM

- Recall that, for PAM,

$$s_m(t) = A_m p(t), \quad 1 \leq m \leq M$$

- Let $\phi(t) = \frac{p(t)}{\sqrt{E_p}}$ where $E_p = \int_{-\infty}^{\infty} |p(t)|^2 \, dt$.
    - Note that $E_\phi = \int_{-\infty}^{\infty} |\phi(t)|^2 \, dt = 1$.
- Then,

$$s_m(t) = A_m \sqrt{E_p} \phi(t) = s^{(m)} \phi(t), \quad 1 \leq m \leq M$$

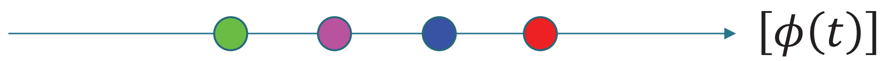





# PAM in the Vector Channel

$$R = S + N$$

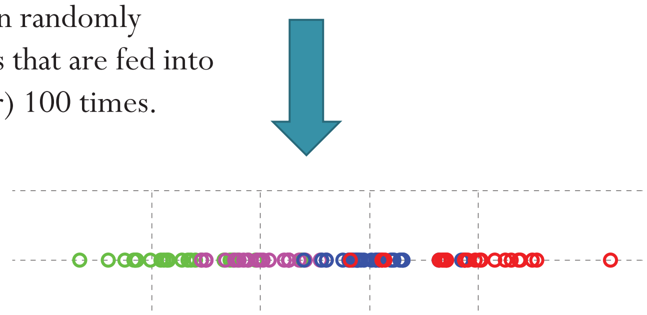
@Tx




The points are chosen randomly (according to the bits that are fed into the digital modulator) 100 times.

Add noise

@ Rx



# Phase-Shift Keying (PSK)

- Digital phase modulation

$$s_m(t) = g(t)\cos(2\pi f_c t + \theta_m), \quad m = 1,2,\ldots,M$$

  - $g(t)$ is the signal pulse shape
  - $\theta_m = \frac{2\pi}{M}(m-1)$ represents phase of the carrier that convey the transmitted information.
- Let

$$\phi_1(t) = \sqrt{\frac{2}{E_g}}\,g(t)\cos(2\pi f_c t) \text{ and } \phi_2(t) = -\sqrt{\frac{2}{E_g}}\,g(t)\sin(2\pi f_c t).$$

  - $\langle \phi_1(t), \phi_2(t) \rangle = 0$ (orthogonal) under appropriate condition
  - $E_{\phi_i} = \int_{-\infty}^{\infty} |\phi_i(t)|^2\,dt = 1.$
- Then,

$$s_m(t) = \sqrt{\frac{E_g}{2}}\cos(\theta_m)\,\phi_1(t) + \sqrt{\frac{E_g}{2}}\sin(\theta_m)\,\phi_2(t).$$



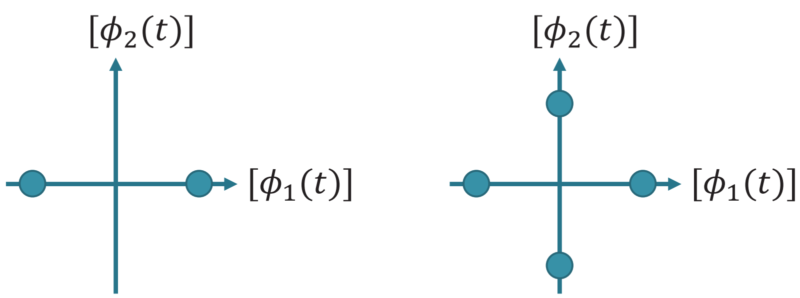
# Constellations for PSK

- Waveform:

$$s_m(t) = \sqrt{\frac{E_g}{2}}\cos(\theta_m)\,\phi_1(t) + \sqrt{\frac{E_g}{2}}\sin(\theta_m)\,\phi_2(t).$$

- Vector:

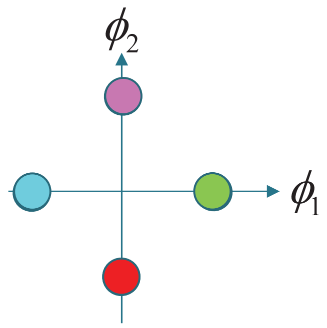
$$\vec{\mathbf{s}}^{(\mathrm{m})} = \left(\sqrt{\frac{E_g}{2}}\cos(\theta_m), \sqrt{\frac{E_g}{2}}\sin(\theta_m)\right).$$

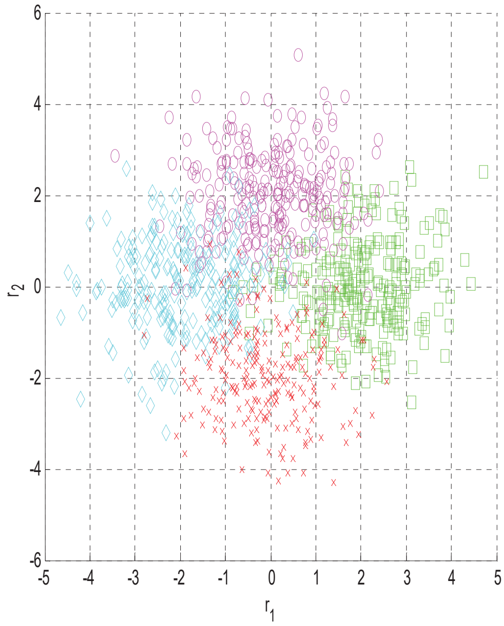



$M = 2$: BPSK

$M = 4$: QPSK

# 4-PSK in the Vector Channel




$$\vec{\mathbf{R}} = \vec{\mathbf{S}} + \vec{\mathbf{N}}$$



# Quadrature Amplitude Modulation (QAM)

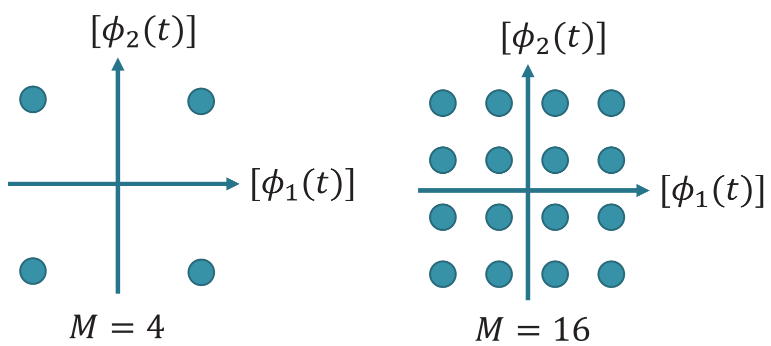
- Waveform:

$$s_m(t) = A_m^{(I)} g(t)\, cos(2\pi f_c t) - A_m^{(Q)} g(t)\, sin(2\pi f_c t)\,, \quad m = 1,2, \ldots, M$$

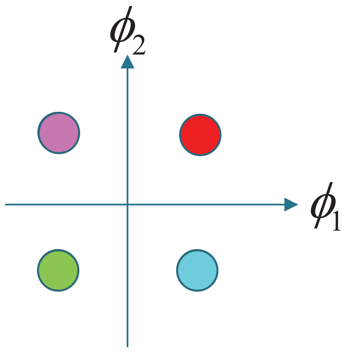
$$= A_m^{(I)} \sqrt{\frac{E_g}{2}} \phi_1(t) + A_m^{(Q)} \sqrt{\frac{E_g}{2}} \phi_2(t)$$

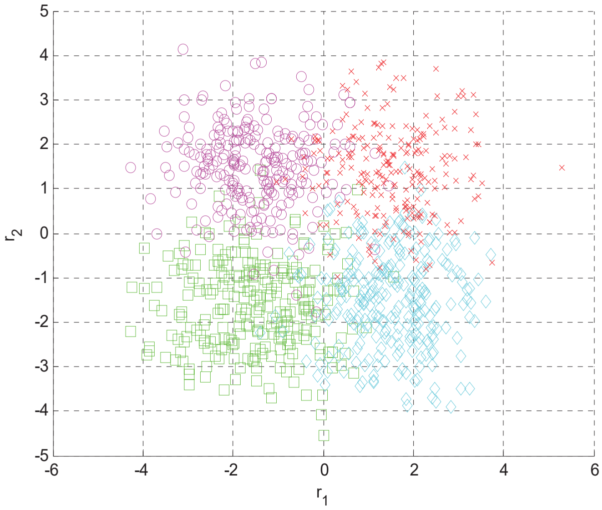
- Vector:

$$\vec{\mathbf{s}}^{(\mathrm{m})} = \left( A_m^{(I)} \sqrt{\frac{E_g}{2}}, A_m^{(Q)} \sqrt{\frac{E_g}{2}} \right).$$



# 4-QAM in the Vector Channel




$$\vec{\mathbf{R}} = \vec{\mathbf{S}} + \vec{\mathbf{N}}$$